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Rainbow Connection Number Of Some Graphs

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Abstract

A rainbow coloring of connected graph is a coloring the edges of graph, such that every pair of vertices is connected by at least one path in which no two edges are colored the same. In this paper, we investigate the rainbow connection number of $\mathbf{P_n + N_m}$ graph, Alternative Double triangular snake graph, quadrilateral snake graph and Subdivision of triangular snake graph.

Keywords— Rainbow vertex coloring, Rainbow coloring, $P_n + N_m$–graph, Triangular Graph.

I. INTRODUCTION

We consider finite, connected and undirected graph. We consider graph $G = (V(G), E(G))$ having set of vertices $V(G)$ and set of edges $E(G)$ respectively. We refer Gross and Yellen [5] for all kind of definitions and notations.

Definition 1.1 In connected graph G, the distance between two of it's vertices v_i and v_j is the length of the shortest path between them. It is denoted by $d(v_i, v_j)$.

Definition 1.2 In a graph G, eccentricity of a vertex $v \in V(G)$, denoted by $E(v)$ is the distance from v to the vertex farthest from v in G i.e. $E(v) = \max_{v_i \in V} d(v, v_i)$.

Definition 1.3 In a graph G, vertex $v \in V(G)$ is called center of G if $E(v)$ is minimum.

Definition 1.4 The diameter of a graph G is denoted by $diam(G)$, defined as $\max_{v_i \in V} E(v_i)$.

Definition 1.5 Edge coloring of a graph G is a function from its edge set to the set of natural numbers.

Definition 1.6 A path in edge colored graph with no two edges having the same color is called rainbow path.

Definition 1.7 An edge colored graph G is called rainbow connected if any two vertices are connected by rainbow path.

Definition 1.8 The rainbow connection number of a graph, denoted by $rc(G)$ is the smallest number of colors that are needed in order to make graph G rainbow connected.

The rainbow connection number was introduced by Chartrand, Johns, McKeon, and Zhang in [4]. D.Parmar ,P. Shah and B.Suthar derived rainbow connection number and Rainbow Connection Number Of Triangular Snake Graph – graph in [3]. Li. Hengzhe and Ma. Yingbin [7] discussed about rainbow connection number and graph operation and Also Annammal and Mercy [2] derives rainbow connection number of shadow graphs. It has applications in transferring information of high security in multicomputer networks. We refer the reader to [8, 9] for details.

Definition 1.9 A Triangular Snake graph T_n is obtained from a path $u_1, u_2, \dots, u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $1 \le i \le n$. That is, every edge of a path is replaced by a triangle.

Definition 1.10 Double Triangular Snake graph $D(T_n)$ consists of two Triangular snakes that have a common path.

Definition 1.11 Alternative Double triangular snake graph from path $u_1, u_2, \dots, u_n$ by joining u_i and u_{i+1} alternatively $(i = 1, 3, 5, \dots)$ to a new vertex $v_1, v_3, \dots \dots, v_{n-1}$ (n is even) or $v_1, v_3, \dots, v_{n-2}$ (n is odd).

Definition 1.12 Quadrilateral snake graph from path $u_1, u_2, \dots, u_n$ by joining u_i and u_{i+1} to two new vertices v_i and v'_i $1 \le i < n$ by considering edge between u_i to v_i, u_{i+1} to v'_i and v_i to v'_i. i.e every edge of a path is replaced by a quadrilateral. Hence we get new vertices $v_1, v_2, \dots, v_{n-1}, v'_1, v'_2, \dots, v'_{n-1}$.

Definition 1.13 [6] Let $P_n + N_m$ be the graph with $|V| = m + n$ and $|E| = 2m + n - 1$ so that $|V \cup E| = 3m + 2n - 1$. $V(P_n + N_m) = \{v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_m\}$ where $V(P_n) = \{v_1, v_2, \dots, v_n\}$ and $V(N_m) = \{u_1, u_2, \dots, u_m\}$,

$$E(P_n + N_m) = E(P_n) \cup \{v_1u_1, v_1u_2, \dots, v_1u_m, v_nu_1, v_nu_2, \dots, v_nu_m\}.$$

In this paper we focus on rainbow vertex connection number and rainbow connection number of $\mathbf{P_n + N_m}$ Graphs, Subdivision of triangular snake graph, quadrilateral snake graph, Alternative Double triangular snake graph. It is easy to see that for any connected graph **G** $\mathbf{diam(G) \le rc(G)}$.

II. MAIN RESULT

Theorem 2.1 Rainbow connection number of $P_n + N_n$ graph of a path P_n is $n-1, n \geq 3$ and even.

Proof: Let $P_n + N_n$ be the graph with $|V| = 2n$ and $|E| = 3n-1$ so that $|V \cup E| = 5n-1$. $V(P_n + N_n) = \{v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n\}$,

where $V(P_n) = \{v_1, v_2, \dots, v_n\}$ and $V(N_n) = \{u_1, u_2, \dots, u_n\}$,

$E(P_n + N_n) = E(P_n) \cup \{v_1u_1, v_1u_2, \dots, v_1 u_n, v_nu_1, v_nu_2, \dots, v_nu_n\}$.

Edge coloring algorithm

Edge	Condition	Color
(v_i, v_{i+1})	$1 \leq i < \frac{n}{2}$	c_{i+1}
(v_i, v_{i+1})	$\frac{n}{2} \leq i < n$	$c_{i-\frac{n}{2}+1}$
(v_1, u_j)	$1 \leq j \leq 2$	c_1
(v_1, u_j)	$1 < j \leq n-1$ (j is odd)	$c_{\frac{j+n-1}{2}}$
(v_1, u_j)	$3 < j \leq n$ (j is even)	$c_{\frac{j+n-2}{2}}$
(v_n, u_j)	$1 < j \leq n-1$ (j is odd)	c_{n-1}
(v_n, u_j)	$3 < j \leq n$ (j is even)	c_{n-2}

We have used total $n-1$ color. Consider any path in $P_n + N_n$– graph. We obtain following cases

Case	Vertex	Vertex	Condition	Rainbow path
1	v_1	v_n	$1 \leq i < j \leq n$	(v_1, u_1, v_n)
2	u_i	u_j	$1 \leq i < j \leq n$	(u_i, v_n, u_j)
3	v_i	u_j	$1 \leq i \leq \frac{n}{2}$, $1 \leq j \leq n$	$(v_i, v_{i-1}, \dots, v_1, u_j)$
4	v_i	u_j	$\frac{n}{2} < i \leq n$, $1 \leq j \leq n$	$(v_i, v_{i+1}, \dots, v_n, u_j)$

Thus, if we consider any path connecting the vertices there exists at least one path in $P_n + N_n$ - graph that form rainbow path. Hence rainbow connection number of $P_n + N_n$- graph is $n-1$.

Illustration 2.2. In figure – 1 shows that Rainbow connection number of $P_n + N_n$–Graph of a path P_6 is 5.

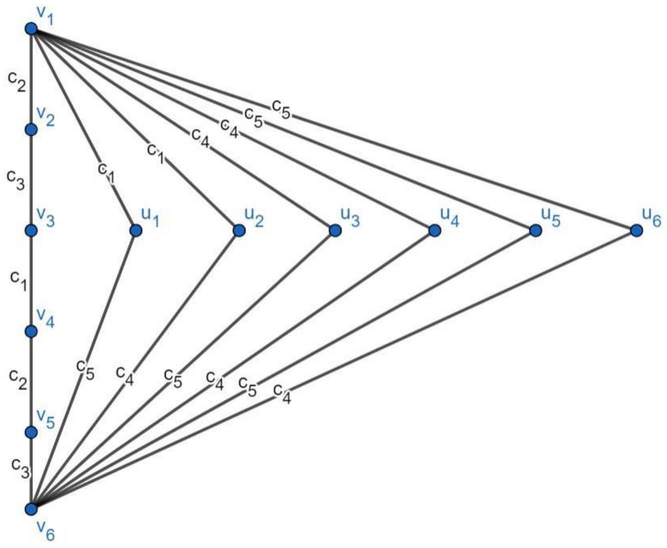

Figure :1

Theorem 2.3. Rainbow connection number of $P_n + N_n$ graph of a path P_n is n , $n \geq 3$ and odd.

Proof: Let $P_n + N_n$ be the graph with $|V| = 2n$ and $|E| = 3n - 1$ so that $|V \cup E| = 5n - 1$. $V(P_n + N_n) = \{v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n\}$,

where $V(P_n) = \{v_1, v_2, \dots, v_n\}$ and $V(N_n) = \{u_1, u_2, \dots, u_n\}$,

$E(P_n + N_n) = E(P_n) \cup \{v_1u_1, v_1u_2, \dots, v_1 u_n, v_nu_1, v_nu_2, \dots, v_nu_n\}$.

Edge coloring algorithm

Edge	Condition	Color
(v_i, v_{i+1})	$1 \leq i < \frac{n+1}{2}$	c_{i+1}
(v_i, v_{i+1})	$\frac{n+1}{2} \leq i < n$	$c_{i-\frac{n+1}{2}+1}$
(v_1, u_j)	$1 \leq j \leq 2$	c_1
(v_1, u_j)	$1 < j \leq n$ (j is odd)	$c_{\frac{j+n}{2}}$
(v_1, u_j)	$3 < j \leq n - 1$ (j is even)	$c_{\frac{j+n-1}{2}}$
(v_n, u_j)	$1 < j \leq n$ (j is odd)	c_{n-1}
(v_n, u_j)	$3 < j \leq n - 1$ (j is even)	c_{n-2}

We have used total n color. Consider any path in $P_n + N_n$– graph. We obtain following case

Case	Vertex	Vertex	Condition	Rainbow path
1	v_1	v_n	-	(v_1, u_1, v_n)
2	u_i	u_j	$1 \leq i < j \leq n$	(u_i, v_n, u_j)

3	v_i	u_j	$1 \le i \le \frac{n+1}{2}, 1 \le j \le n$	$(v_i, v_{i-1}, \ldots, v_1, u_j)$
4	v_i	u_j	$\frac{n+1}{2} < i \le n, 1 \le j \le n$	$(v_i, v_{i+1}, \ldots, v_n, u_j)$

Thus, if we consider any path connecting the vertices there exists at least one path in $P_n + N_n$ - graph that form rainbow path. Hence rainbow connection number of $P_n + N_n$- graph is n.

llustration 2.4. In figure – 2 shows that Rainbow connection number of $P_n + N_n$–Graph of a path P_7 is 7.

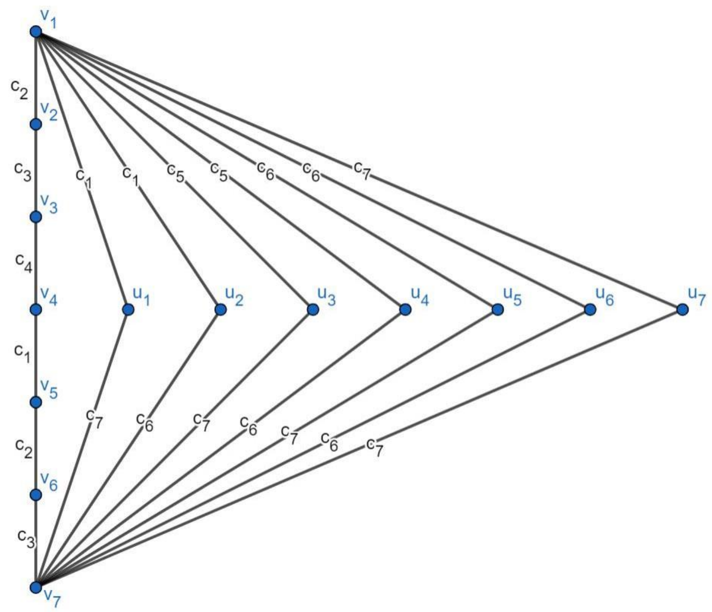

Figure :2

Theorem 2.5. Rainbow connection number of $P_n + N_{n+1}$ graph of a path P_n is n, $n \ge 3$.

Proof: Let $P_n + N_{n+1}$ be the graph with $|V| = 2n + 1$ and $|E| = 3n + 1$ so that $|V \cup E| = 3(n + 1) + 2n - 1$. $V(P_n + N_{n+1}) = \{v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_{n+1}\}$,

where $V(P_n) = \{v_1, v_2, \ldots, v_n\}$ and $V(N_{n+1}) = \{u_1, u_2, \ldots, u_{n+1}\}$,

$E(P_n + N_{n+1}) = E(P_n) \cup \{v_1u_1, v_1u_2, \ldots, v_1 u_{n+1}, v_nu_1, v_nu_2, \ldots, v_nu_{n+1}\}$. $E(P_n + N_{(n-1)}) = E(P_n) \cup \{v_1u_1, v_1u_2, \ldots, v_1 u_{(n-1)}, v_nu_1, v_nu_2, \ldots, v_nu_{(n-1)}\}$.Edge coloring algorithm: If $n \ge 3$

Edge	Condition	Color
(v_i, v_{i+1})	$1 \le i < \frac{n}{2}$, (if n is even) $1 \le i < \frac{n+1}{2}$, (if n is odd)	c_{i+1}

(v_i, v_{i+1})	$\frac{n}{2} \le i < n$, (if n is even) $\frac{n+1}{2} \le i < n$, (if n is odd)	$c_{i-\frac{n}{2}+1}$, (if n is even) $c_{i-\frac{n+1}{2}+1}$, (if n is odd)
(v_1, u_j)	$1 \le j \le 2$	c_1
(v_1, u_j)	$1 < j \le n$, (j is odd & n is odd) $1 < j \le n-1$ (j is odd & n is even)	$c_{\frac{j+n}{2}}$ $c_{\frac{j+n-1}{2}}$
(v_1, u_j)	$3 < j \le n-1$ (j is even & n is odd) $3 < j \le n$ (j is even & n is even	$c_{\frac{j+n-1}{2}}$ $c_{\frac{j+n-2}{2}}$
(v_n, u_j)	$1 < j \le n$ (j is odd)	c_{n-1}
(v_n, u_j)	$3 < j \le n-1$ (j is even)	c_{n-2}

We have used total n color. Consider any path in $P_n + N_{n+1}$– graph. We obtain following cases

Case	Vertex	Vertex	Condition	Rainbow path
1	v_1	v_n	-	(v_1, u_1, v_n)
2	u_i	u_j	$1 \le i < j \le n+1$	(u_i, v_n, u_j)
3	v_i	u_j	$1 \le i \le \frac{n}{2}$, $1 \le j \le n+1$, (if n is even) $1 \le i \le \frac{n+1}{2}$, $1 \le j \le n+1$, (if n is odd)	$(v_i, v_{i-1}, \ldots, v_1, u_j)$
4	v_i	u_j	$\frac{n}{2} < i \le n$, $1 \le j \le n$, (if n is even) $\frac{n+1}{2} < i \le n$, $1 \le j \le n$, (if n is odd)	$(v_i, v_{i+1}, \ldots, v_n, u_j)$

Thus, if we consider any path connecting the vertices there exists at least one path in $P_n + N_{n+1}$- graph that form rainbow path. Hence rainbow connection number of $P_n + N_{n+1}$ graph is n.

Illustration 2.6. In figure – 3 shows that Rainbow connection number of $P_5 + N_6$–Graph of a path P_5 is 5.

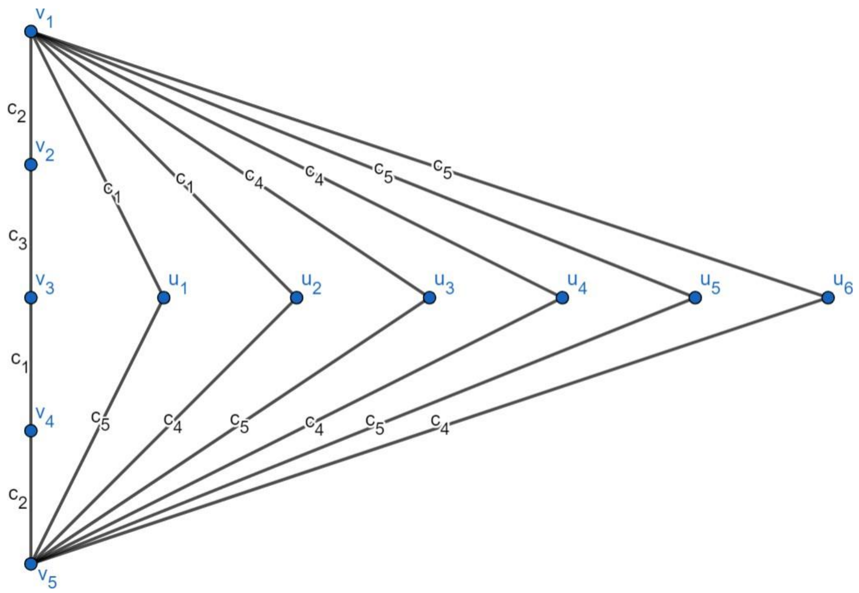

Figure :3

Theorem 2.7. Rainbow connection number of $P_n + N_{n-1}$ graph of a path P_n is $n-1, n \geq 3$. Proof: Let

$P_n + N_{n-1}$ be the graph with $|V| = (n-1) + n$ and $|E| = 2(n-1) + n - 1$
so that $|V \cup E| = 3(n-1) + 2n - 1$. $V(P_n + N_{(n-1)}) = \{v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_{(n-1)}\}$,

where $V(P_n) = \{v_1, v_2, \ldots, v_n\}$ and $V(N_{(n-1)}) = \{u_1, u_2, \ldots, u_{(n-1)}\}$,

$E(P_n + N_{(n-1)}) = E(P_n) \cup \{v_1u_1, v_1u_2, \ldots, v_1 u_{(n-1)}, v_nu_1, v_nu_2, \ldots, v_nu_{(n-1)}\}$.

Edge coloring algorithm: If $n \geq 3$

Edge	Condition	Color
(v_i, v_{i+1})	$1 \leq i < \frac{n}{2}$, (if n is even) $1 \leq i < \frac{n+1}{2}$, (if n is odd)	c_{i+1}
(v_i, v_{i+1})	$\frac{n}{2} \leq i < n$ $\frac{n+1}{2} \leq i < n$	$c_{i-\frac{n}{2}+1}$, (if n is even) $c_{i-\frac{n+1}{2}+1}$, (if n is odd)
(v_1, u_j)	$1 \leq j \leq 2$	c_1
(v_1, u_j)	$1 < j \leq n$ (j is odd & n is odd) $1 < j \leq n - 1$ (j is odd & n is even)	$c_{\frac{j+n}{2}}$ $c_{\frac{j+n-1}{2}}$
(v_1, u_j)	$3 < j \leq n-1$ (j is even & n is odd) $3 < j \leq n$ (j is even & n is even)	$c_{\frac{j+n-1}{2}}$ $c_{\frac{j+n-2}{2}}$
(v_n, u_j)	$1 < j \leq n$ (j is odd)	c_{n-1}

(v_n, u_j)	$3 < j \leq n - 1$ (j is even)	c_{n-2}

We have used total n color. Consider any path in $P_n + N_{n-1}$– graph. We obtain following cases

Case	Vertex	Vertex	Condition	Rainbow path
1	v_1	v_n	-	(v_1, u_1, v_n)
2	u_i	u_j	$1 \leq i < j \leq n + 1$	(u_i, v_n, u_j)
3	v_i	u_j	$1 \leq i \leq \frac{n}{2}, 1 \leq j \leq n + 1$, (if n is even) $1 \leq i \leq \frac{n+1}{2}, 1 \leq j \leq n + 1$, (if n is odd)	$(v_i, v_{i-1}, \ldots, v_1, u_j)$
4	v_i	u_j	$\frac{n}{2} < i \leq n, 1 \leq j \leq n$, (if n is even) $\frac{n+1}{2} < i \leq n, 1 \leq j \leq n$, (if n is odd)	$(v_i, v_{i+1}, \ldots, v_n, u_j)$

Thus, if we consider any path connecting the vertices there exists at least one path in

$P_n + N_{n-1}$- graph that form rainbow path. Hence rainbow connection number of

$P_n + N_{n-1}$ graph is $n - 1$.

Illustration 2.8. In figure – 4 shows that Rainbow connection number of $P_5 + N_4$–Graph of a path P_5 is 4.

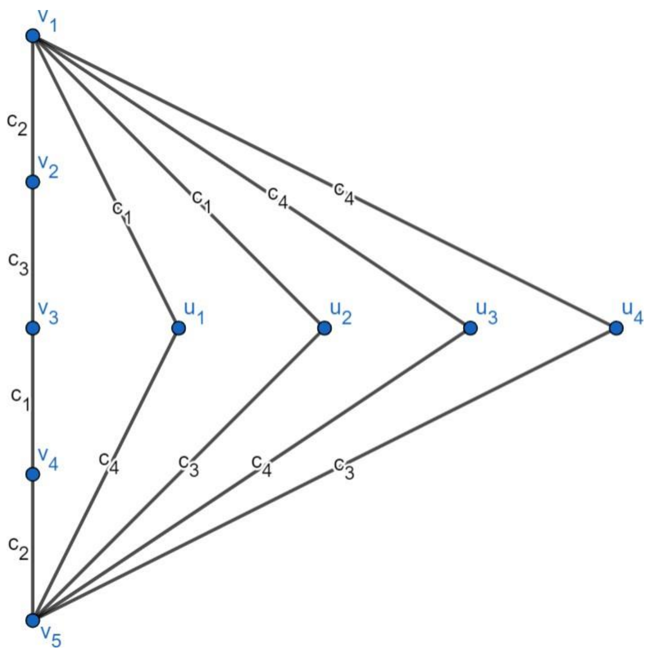

Figure :4

Theorem 2.9 Rainbow connection number Double Alternative triangular snake graph $rc(D(A(T_n)))$ is $n-1$, $n \geq 2$.

Proof: Let P_n be the path $u_1, u_2, \ldots, u_n$. Then it is clear that $diam\,(D(A(T_n)) = n-1$.

We can obtain Alternative Double triangular snake graph from path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} alternatively $(i = 1, 3, 5, \ldots)$ to a new vertex $v_1, v_3, \ldots, v_{\frac{n}{2}}$ (n is even) or $v_1, v_3, \ldots, v_{\frac{(n-1)}{2}}$ (n is odd).

Edge coloring algorithm:

Edge	Condition	Color
(u_i, u_{i+1})	$1 \leq i < n$	$c_1, c_2, \ldots, c_{n-1}$
(u_i, v_i) and (u_{i+1}, v_i)	$i = 1, 3, 5, \ldots$ and $i < n$	c_i
(u_i, v'_i)	$i = 1,3,5, \ldots$ and $i < n$.	$c_{\left(\frac{n}{2}\right)}$ (n is even)
(u_i, v'_i)	$i = 1,3,5, \ldots$ and $i < n$.	$c_{\left(\frac{n+1}{2}\right)}$ (n is odd)
(u_{i+1}, v'_i)	$i = 1, 3, 5, \ldots$ and $i < n$	c_1

Consider any path in $(D(A(T_n)))$, We get following cases

Case	Vertex	Vertex	Condition	Rainbow path
1	u_i	u_j	$1 \leq i < j \leq n$	$u_i, u_{i+1}, \ldots, u_j$
2	v_i	v_j	$1 \leq j < i \leq n$	$v_i, u_{i+1}, \ldots, u_j, v_j$
3	u_i	v_j	$1 \leq i \leq n, 1 \leq j \leq n$	$\begin{cases} u_i, u_{i+1}, \ldots, u_j, v_j, & i < j \\ u_i, u_{i-1}, \ldots, u_{j+1}, v_j, & i > j \end{cases}$
4	u_i	v'_j	$1 \leq i < j < n$	$u_i, u_{i+1}, \ldots, u_j, v'_j$
5	v_i	v'_j	$1 \leq i < j < n$	$v_i, u_{i+1}, \ldots, u_j, v'_j$
6	v'_i	v'_j	$1 \leq i < j < n$	$v'_i, u_{i+1}, \ldots, u_j, v'_j$

Thus, if we consider any path connecting the vertices there exists at least one path in $D(A(T_n))$ that form rainbow path. Hence $rc(D(A(T_n)) = n-1$.

Illustration 2.9, In figure – 5 shows that Rainbow connection number Double Alternative triangular snake graph $rc(D(A(T_6)) = 5$.

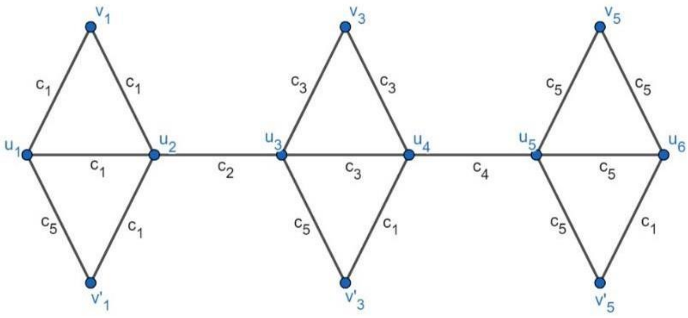

figure – 5

Theorem 2.10. Rainbow Connection number Quadrilateral snake graph $rc(Q_n)$ is $n + 1,\ n \geq 2$.

Proof: Let P_n be the path $u_1, u_2, \dots, u_n$. Then it is clear that $diam\,(Q_n) = n + 1$.

We can obtain quadrilateral snake graph from path $u_1, u_2, \dots, u_n$ by joining u_i and u_{i+1} to two new vertices v_i and v'_i $1 \leq i < n$ by considering edge between u_i to v_i, u_{i+1} to v'_i and v_i to v'_i. i.e every edge of a path is replaced by a quadrilateral.

Hence we get new vertices $v_1, v_2, \dots, v_{n-1}, v'_1, v'_2, \dots, v'_{n-1}$.

Edge coloring algorithm:

Edge	Condition	Color
(u_i, u_{i+1})	$1 \leq i \leq n$	$c_1, c_2, \dots, c_{n-1}$
(u_i, v_i) and (u_{i+1}, v'_i)	$1 \leq i < n$	c_n and c_{n+1}
(v_i, v'_i)	$1 \leq i < n$	c_i

Consider any path in $rc(Q_n)$, We get following cases

Case	Vertex	Vertex	Condition	Rainbow path
1	u_i	u_j	$1 \leq i < j \leq n$	$u_i, u_{i+1}, \dots, u_j$
2	v_i	v_j	$1 \leq j < i \leq n$	$v_i, v'_i, u_{i+1}, \dots, u_j, v_j$
3	v'_i	v'_j	$1 \leq i < j < n$	$v'_i, u_{i+1}, \dots, u_j, v_j, v'_j$
4	u_i	v_j	$1 \leq i < j < n$	$u_i, u_{i+1}, \dots, u_j, v_j$
5	u_i	v'_j	$1 \leq i < j < n$	$u_i, u_{i+1}, \dots, u_j, v'_j$
6	v_i	v'_j	$1 \leq i < j < n$	$v_i, v'_i, u_{i+1}, \dots, u_{j+1}, v'_j$

Thus, if we consider any path connecting the vertices there exists at least one path in Q_n that form rainbow path. Hence $rc(Q_n) = n + 1$.

Illustration 2.11. In figure – 6 shows that Rainbow connection number quadrilateral snake graph $rc(Q_6) = 7$.

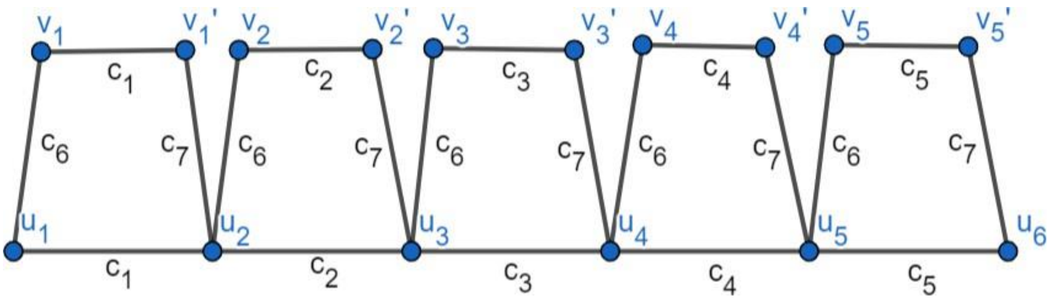

Figure :6

Theorem 2.12 Rainbow connection number of subdivision triangular snake graph $rc(S(T_n)) = 2n$, $n \geq 3$.

Proof: Let P_n be the path $u_1, u_2, \ldots, u_n$. Then it is clear that $diam\, S(T_n) = 2n$.

We can obtain triangular snake graph from path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} to a new vertex v_i, $1 \leq i < n$.

Take subdivision of each edge of triangular snake graph by introducing new vertices $w_i, w'_i, w''_i, 1 \leq i \leq n-1$.

Hence $V(S(T_n)) = u_i, v_j, w_i, w'_i, w''_i, 1 \leq i \leq n, 1 \leq j \leq n-1$.

And $E(S(T_n)) = u_iw_i, w_iu_{i+1}, u_iw'_i, w'_iv_i, v_iw''_i, w''_iu_{i+1}, 1 \leq i \leq n-1$.

Edge coloring algorithm:

Edge	Condition	Color
(u_i, w_i) and (w_i, u_{i+1})		c_{2i-1} and c_{2i}
(u_i, w'_i)		c_{2n-1}
(w_i, u_{i+1})	$1 \leq i \leq n-1$	c_{2n}
(w'_i, v_i)		c_{2i}
(v_i, w''_i)		c_{2i-1}

Consider any path in $S(T_n)$.

Case	Vertex	Vertex	Condition	Rainbow path

1	u_i	u_j	$1 \le i < j \le n$	$u_i, w_i, u_{i+1}, \ldots, u_j$
2	v_i	v_j	$1 \le i < j \le n-1$	$v_i, w_i, u_{i+1}, w_{i+1}, u_{i+2}, \ldots, w_j, v_j$
3	u_i	w_j	$1 \le i < j \le n$	$u_i, w_i, u_{i+1}, \ldots, u_j, w_j$
4	u_i	w'_j	$1 \le i < j \le n-1$	$u_i, w_i, u_{i+1}, \ldots, u_j, w'_j$
5	u_i	v_j	$1 \le i < j \le n-1$	$u_i, w_i, u_{i+1}, \ldots, w_j, v_j$
6	u_i	w''_j	$1 \le i < j \le n-1$	$u_i, w_i, u_{i+1}, \ldots, w_j, u_{j+1}, w''_j$
7	v_i	w'_j	$1 \le i < j \le n-1$	$u_i, w''_i, u_{i+1}, \ldots, u_j, w'_j$
8	v_i	w_j	$1 \le i < j \le n-1$	$v_i, w''_i, u_{i+1}, \ldots, u_j, w_j$
9	v_i	w''_j	$1 \le i < j \le n-1$	$v_i, w''_i, u_{i+1}, \ldots, u_{j-1}, w_j, u_{j+1}, w''_j$
10	w_i	w_j	$1 \le i < j \le n$	$w_i, u_{i+1}, \ldots, u_{j-1}, w_j$
11	w_i	w''_j	$1 \le i < j \le n-1$	$w_i, u_{i+1}, \ldots, w_j, u_{j+1}, w_j$
12	w_i	w'_j	$1 \le i < j \le n-1$	$w_i, u_{i+1}, \ldots, u_j, w'_j$
13	w'_i	w'_j	$1 \le i < j \le n-1$	$w'_i, u_i, w_i, u_{i+1}, \ldots, u_j, w'_j$
14	w'_i	w''_j	$1 \le i < j \le n-1$	$w'_i, u_i, w_i, u_{i+1}, \ldots, u_{j+1}, w''_j$
15	w''_i	w''_j	$1 \le i < j \le n-1$	$w''_i, u_{i+1}, w_{i+1}, \ldots, u_{j+1}, w''_j$

Thus, if we consider any path connecting the vertices there exists at least one path in $S(T_n)$ that form rainbow path. Hence $rc(S(T_n)) = 2n$.

Illustration 2.13 In figure – 7 shows that Rainbow connection number of subdivision triangular snake graph $rc(S(T_6)) = 12$.

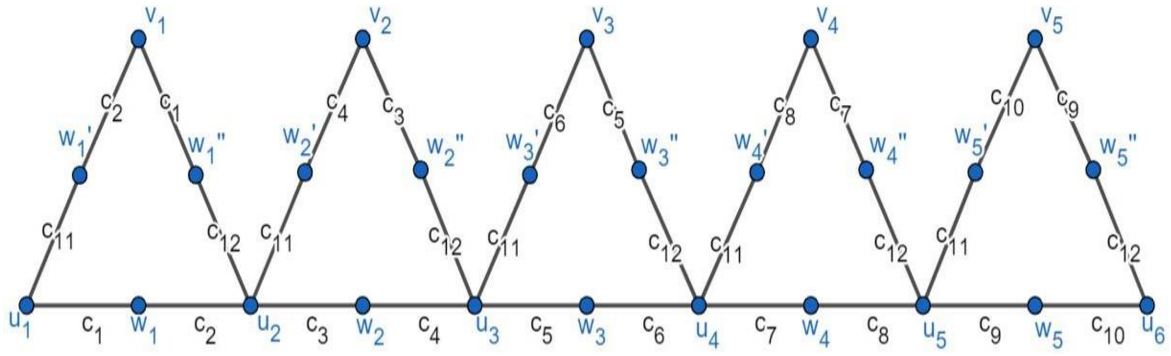

Figure :7

III. CONCLUSION

The rainbow coloring of $P_n + N_m$ graph, Alternative Double triangular snake graph, quadrilateral snake graph and Subdivision of triangular snake graph has been defined and their rainbow connection numbers have been computed using rainbow edge coloring.

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